Response to "Comment on 'Turbulent equipartition theory of toroidal momentum pinch' " [Phys. Plasmas 16, 034703 (2009)]

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(Received 15 May 2008; accepted 18 February 2009; published online 25 March 2009)

This response demonstrates that the comment by Peeters *et al.* contains an incorrect and misleading interpretation of our paper [T. S. Hahm *et al.*, Phys. Plasmas **15**, 055902 (2008)] regarding the density gradient dependence of momentum pinch and the turbulent equipartition theory.
 [DOI: 10.1063/1.3096714]

I. INTRODUCTION

The subject discussed in this reply is in relation to the recent theoretical predictions of a parallel momentum pinch in toroidal geometry. The main purpose of our recent paper[1] was to elucidate the physics associated with that pinch in the context of the turbulent equipartition (TEP) theory, which relies on local conservation properties, and mixing of a magnetically weighted quantity. Following upon a rather lengthy derivation[2] based on the gyrokinetic equation which includes both the TEP pinch which is mode independent, and the curvature driven thermoelectric (CTh) pinch which is mode dependent, we reported a more concise physics-oriented interpretation of the TEP pinch.[1,3] We have formulated the problem in terms of the angular momentum density based on a conservative gyrokinetic equation in general toroidal geometry.[4] On the other hand, Peeters *et al.*,[5] formulated the pinch in terms of the Coriolis force which appears in the rotating frame. The actual analytic derivation has been performed in a shearless slab geometry, but with the Coriolis and magnetic drifts kept with the purpose to derive a momentum pinch for a pure ion temperature gradient (ITG) mode with an adiabatic electron response. They dealt with the parallel flow (without a density multiplier). With these differences in approaches, emphases, and also in ensuing approximations, there have been some misunderstandings and improper interpretations of our work and their own work in their comment.[6]

The comment by Peeters *et al.*,[6] contains the following claims with which we disagree based on technical grounds:

(i) The linear dependence on density gradient length, in the pinch to diffusivity ratio expression in their letter,[5] comes from the Coriolis drift;
(ii) their result for the ITG mode with an adiabatic electron response contains our TEP formulation.

Their comment[6] also includes a figure comparing the pinch to diffusivity ratio from a numerical calculation, from a reformulation of their formula, and from our original expression. Since those were obtained under different assumptions and for different physical models, in particular, inclusion of trapped electrons, and parameters, we find their comparison to be misleading. We clarify these issues in this reply.

II. MAIN DIFFERENCE BETWEEN OUR PAPERS AND THEIR LETTER

The main focus of our paper on the TEP momentum pinch[1] was on physics understanding which is independent of the particle flux model and also of the specific modes such as the ITG or trapped electron mode (TEM). Our preceding paper illustrated details of derivations from the gyrokinetic equation.[2] Throughout the derivation, manifestations of new terms, which came from the original conservative gyrokinetic equations,[4] have been carefully studied and discussed. It's crucial to remember that we have identified the "CTh pinch" in addition to the TEP pinch. This CTh pinch is proportional to the correlation between the ion temperature fluctuation (δT_i) and the potential fluctuation ($\delta\phi$) weighted by the magnetic curvature and grad-B drift, ω_{di} [see Eqs. (11), (39), and (66) of Ref. 2]. We have not claimed that our TEP pinch is the full story (see conclusions in Refs. 1 and 2), but emphasized that the TEP pinch depends only on the fluctuation amplitude and the decorrelation time, and is a mode-independent generic and robust feature (i.e., whether it is ITG or TEM).

On the other hand, in the derivation of Peeters *et al.*,[5] intended for the pure ITG mode, the ion temperature evolution equation [Eq. (15) in Ref. 5] has *never* been used. Their derivation did not properly take into account a dynamical role of the ion temperature fluctuations, and therefore, could not describe physics associated with the CTh pinch in a realistic system.

Our theory is also formulated in general toroidal geometry, which allows shaping, with geodesic curvature, magnetic shear, etc., and did not make an assumption on the temperature ratio T_i/T_e. The effect of mode structure on the final answer has also been characterized by a dimensionless parameter F_{balloon}, which was calculated for parameters from actual experiments.[2] This is unlike the letter by Peeters *et al.*,[5] where a local analysis has been performed with a fictitious Coriolis drift and magnetic drift in a system which is Galilean invariant in the direction of flow. They have also

TABLE I. Comparison of different approaches.

	Hahm, Diamond, Gurcan, and Rewoldt (Ref. 2)	Gurcan *et al.* (Ref. 3); Hahm *et al.* (Ref. 1)	Peeters *et al.* (Ref. 5)
Starting equation	Gyrokinetic equation in laboratory (Hahm 88; Ref. 4)	Local angular momentum density conservation	Gyrokinetic equation in rotating frame (Brizard 95; Ref. 8)
Effects of mode structure	Characterized by F_{balloon}; calculated by FULL code for exemplary case	Characterized by F_{balloon};	Local analysis
Instability considered in analytic theory	Any long-wavelength instabilities on gyroradius scale; mode-dependent calculation ends with general expression which has not been pursued to the end	Only mode-independent part of pinch: TEP pinch (TEP) has been considered	Long-wavelength ITG instability with adiabatic electrons
Physics mechanisms for momentum pinch	Magnetic curvature modification to parallel acceleration, leading to B^* symmetry breaking	Homogenization (mixing) of magnetically weighted angular momentum density	Coriolis-force-driven drift coupling density, flow, and temperature fluctuations
Physics of TEP pinch	TEP part of pinch identified from GK derivations, discussed in the context of homogenization theory	Presented starting from local conservation laws and quasilinear theory of magnetically weighted angular momentum density	Not discussed

assumed equal temperatures, $T_i=T_e$. If one relaxes this condition, for a more direct comparison with our prediction, the derivation according to Peeters *et al.*[5] would have led to a formula $V_{\text{pinch}}/\chi_\phi=-1/L_n-4T_i/T_eR$, which is different from our TEP prediction,[1,2] which is independent of T_i/T_e. For these reasons, it must be obvious that the TEP pinch *cannot* simply be a subset of their solution, as Peeters *et al.* claim based on the fact that they kept more terms. To set the record straight, we would like to mention that neither the phrase "TEP" nor the phrase "symmetry breaking mechanism" was ever used in their letter.[5] While they state that their simple analytic fluid model is for highlighting the physics effect, it is noteworthy that the Coriolis drift and the ballooning mode structure are incompatible with a system with Galilean invariance along a straight magnetic field. Table I summarizes the main difference between our papers and their letter. Finally, the comment of Peeters *et al.*[6] contains an incorrect description of our TEP theory for the momentum pinch, stating that "Hahm *et al.* assume mixing of the toroidal angular momentum," while we did not.

III. ON THE SCALING OF THE PINCH TO DIFFUSIVITY RATIO FROM ANALYTIC MODELS

We have stated[1] that the $1/L_n$ term in the analytic formula from Ref. 5, i.e., $V_{\text{pinch}}/\chi_\phi=-1/L_n-4/R$, cannot be attributed to the Coriolis drift, which vanishes in the limit $1/R\to 0$. If such a linear dependence on L_n^{-1} is real, it should come from a different physics mechanism. Note that we did not claim that a L_n^{-1} dependence is impossible, in general. The comment by Peeters *et al.* lists various dimensionless quantities which appear in the linear gyrokinetic equation (including R/L_n), and argues that their pinch to diffusivity ratio formula from a simple fluid theory does not contradict the general properties of the linear gyrokinetic equation. While their result is dimensionally correct, their analytic derivation neglected wave particle resonances including the one involving the magnetic drift, and additionally local toroidicity ($\epsilon=r/R_0$), magnetic shear and geodesic curvature, and any relation of their result to the general properties of the gyrokinetic equation in toroidal geometry is already disconnected. Then, what is the origin of that $1/L_n$ dependence in their formula? They used the ion density continuity equation [Eq. (13)] rather than the ion temperature evolution equation [Eq. (15)]. Restoring dimensions to various frequencies to make our physical argument clear, their derivation is based on a dominant balance between $(\omega-\omega_{*e})\delta\phi$ and $-2\omega_{di}(\delta T_i+2\delta\phi)$. The density gradient dependence comes from the $\omega_{*e}\delta\phi$ term. If we take this relation seriously, $\delta\phi$ should be more strongly ballooning at the low field side compared to δT_i since the magnetic curvature related ω_{di} is the origin of the ballooning structure. Unfortunately this is quite contrary to the results of fully developed ITG turbulence in toroidal geometry which indicate that δT_i balloons more strongly than $\delta\phi$.[7] This shows that their simple analytic model has

artifacts which should be taken with great caution when it is applied to comparisons to experiments or projection to larger machines. Comparing a numerical result from a linear gyrokinetic calculation with simple analytic formulas in Fig. 1, without stating important differences in physical models, assumptions and parameters, is misleading. For instance, they do not specify the equation for "kinetic electrons." Therefore, as presented, it is not even clear what problem has been solved numerically.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy through Contract No. DE–AC02–76–CHO-3073 (PPPL) and Grant No. FG02-04ER 54738 (UCSD).

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